Verification of a Novel 2D Finite Element for Reinforced and Prestressed Concrete

Mishael E. Nuh[1], Edvard P. G. Bruun[2], Evan C. Bentz[2]

The Modified Compression Field Theory (MCFT) [1] can be implemented in finite element programs [2] to analyze complex reinforced and prestressed concrete structures, such as offshore oil platforms and nuclear containment silos. However, typical finite element approaches are faced with a trade-off between large computational times and the accuracy of the results. A unique 2D finite element based on the MCFT, developed at the University of Toronto, strives to safely predict the behaviour of reinforced concrete structures subjected to in-plane loading at a significantly reduced computational cost. This research project aims to provide numerical verification of the 2D element. Two peer-reviewed databases totalling over 800 shear critical beam tests [3][4] were used to measure the element's ability to respond to shear failure—a difficult mode of failure to predict. The databases contains point load tests of rectangular and T-beams of varying sizes and material properties. To simulate the experiments, finite element models based on the element were created, the results of which were then compared to the reported values. As the speed and accuracy of the model is influenced by the number of elements along the beam, a mesh sensitivity analysis was performed on select beams to characterize the impact of the element size. It was found that the model was generally able to conservatively predict the failure of concrete beams, and that the modelling was robust with respect to the element size. Generalizing this 2D model into a 3D model to capture the behavior of concrete members subjected to in-plane and out-of-plane loading will enable researchers and design engineers to quickly and accurately verify the safety of newly designed structures subject to more complex loading.

Introduction

Reinforced and prestressed concrete has become ubiquitous as a building material due to its low cost and versatility. However, its behaviour and response to various loadings were initially not properly understood, and much research is still ongoing to better characterize this material. This is especially true of shear failure—a challenging mode of failure to predict. While factors of safety are included to reduce the impact of an unconservative prediction, recent failures such as the collapse of the De la Concorde overpass in 2006 due to shear failure [5] show that there is still a need for tools to verify the structural integrity of newly designed and pre-existing structures.

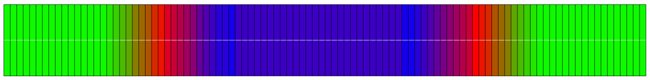
Finite element modelling offers a solution by enabling engineers to simulate complex structures under different loading conditions. By discretizing the structure into a mesh of smaller elements, the overall behaviour can be predicted from the response of its constituent parts. It is a financially low cost and versatile method, which makes it suitable for use in the analysis of a variety of structures such as offshore oil platforms and nuclear containment silos that would be too expensive to physically test. However, larger and more complex structures require more elements to model, which increases the computational cost of the analysis. Different types of finite elements, such as shell elements, which can represent 3D effects [6], or planar elements, which only interact with 2D forces, can be implemented as needed to reduce the number of elements. Nevertheless, the aim is often not specifically towards reducing computational times, but rather towards increasing the accuracy of the model.

A novel 2D element based on the MCFT, developed at the University of Toronto, aims to reduce computational costs while also conservatively predicting the behaviour of reinforced and prestressed concrete structures. This project aims to verify the accuracy of the element and to investigate the relationship between the number of elements and the required computational time and accuracy of the results.

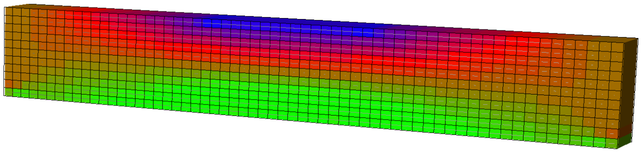
Methodology

Utilizing a shear panel implemented using the MCFT is a typical way of constructing a finite element model for reinforced concrete. This concept has been implemented

[1]Division of Engineering Science, University of Toronto
[2]Department of Civil Engineering, University of Toronto

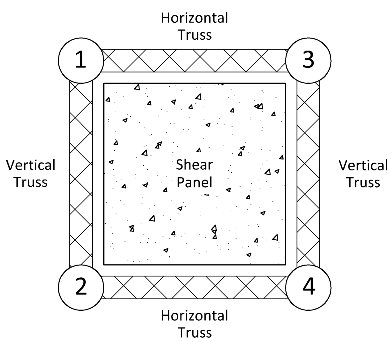


(a) Novel finite element model

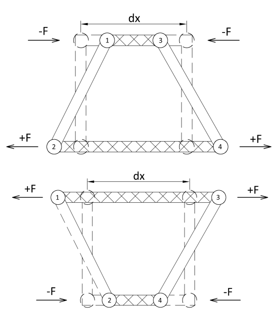


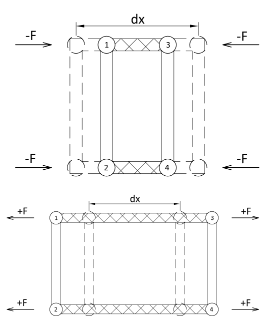
(b) Typical finite element model

Fig. 1. Finite element mesh and results of a beam modeled using two different finite element models: (a) uses the novel 2D element with one element over the entire depth, while (b) uses a typical finite element model.

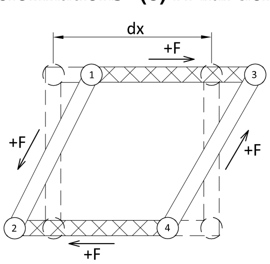


(a) Element





(b) Flexural deformations **(c)** Axial deformations



(d) Shear deformations

Fig. 2. Schematic of the novel finite element showing (a) the parts of the elements, (b) flexural deformations, (c) axial deformations, and (d) shear deformations.

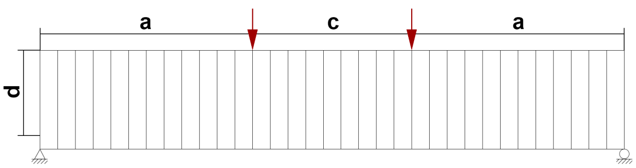
successfully at the University of Toronto through non-linear finite element analysis programs such as those in the VecTor suite, with VecTor2 implementing the shear panel specifically for a 2D element [2]. But as the panel is only able to capture shear deformations, multiple elements must be used along the depth of the beam to properly capture flexural and axial deformations. Building upon this concept of the shear panel, the novel element adds four trusses which surrounds the panel to explicitly capture flexural and axial deformations as well. Figure 2 shows how the element is able to deform in a variety of manners as a result of this addition. By explicitly capturing both shear and flexural deformations, the novel finite element allows one element to be placed over the entire depth of a beam as seen in Figure 1, thereby significantly reducing the overall number of elements required to model the beam. In addition, since the axial trusses are built into the element formulation, their parameters do not have to be specified by the user when constructing the model. Thus, the novel element is relatively simple to use and its added features does not translate into additional complexity for a user in its implementation.

Verification

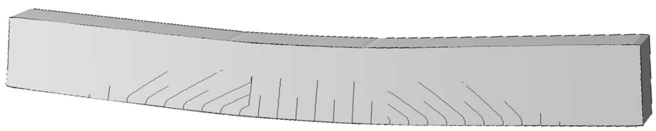
Two peer-reviewed databases containing point-load tests of shear critical beams [3] [4] were used to measure the element's ability to predict shear failure. One database contained beams with shear reinforcements, while the other did not. A number of tests were excluded as they contained incorrect or missing values which were necessary for modelling the beams, resulting in a total of 874 tests. The model input files were generated from the database with various parameters such as the dimensions of the cross section, the concrete compressive strength, and the yield strength of the reinforcing steel. Figure 3a shows a beam modeled using the element along with the location of the point loads. The beams were sectioned along its length into individual elements with a target width of 25 mm and the minimum number of elements for each beam was set to 25 with a maximum of 100. This is a conservative modelling parameter as it was found that only 5 elements are needed across the shear span (a in Figure 3a) to accurately model a beam. Each model was then processed through the Augustus-II software package developed at the University of Toronto which has implemented the finite element solution procedure. The load at failure was recorded and compared with the experimental test value reported in the database. The ratio between the two values (the Exper./Pred. ratio) was used as the main metric for assessing the accuracy of the model.

Results

The average Exper./Pred. ratio was found to be 1.39 with a coefficient of variation of 21.8%. Figure 4 shows the distribution of the Exper./Pred. ratio for both beams with stirrups (shear reinforcement) and no stirrups. One



(a) Beam modeled in Augustus-II



(b) Beam cracking during analysis

Fig. 3. A typical beam modeled using the novel finite element model. Important dimensions are labeled and location of point loads are indicated by arrows

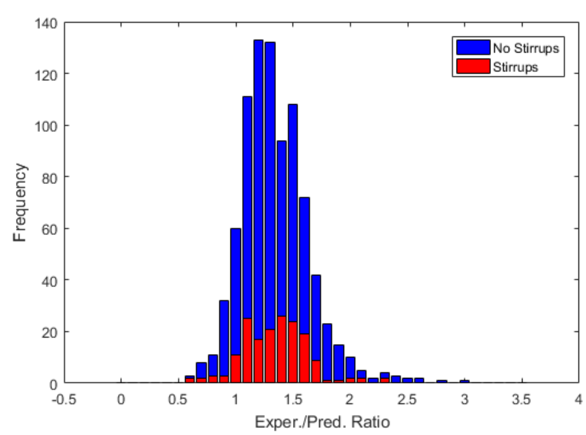


Fig. 4. Exper./Pred. ratio distribution for beams with and without shear reinforcement.

contributing factor to the high average Exper./Pred. ratio can be the material inconsistencies and imperfections in the manufacturing of smaller beams. In addition, many beams in the database are detailed in a way that is not representative of realistic members. This will be further investigated in the *Discussion* section of the paper.

Mesh Sensitivity Analysis

In order to characterize how the failure value converges with respect to the number of elements, a mesh sensitivity analysis was performed. Starting with a small value, the number of elements was slowly incremented and the Exper./Pred. ratios were recorded. Beams #131 and #1358 from the databases were randomly chosen for this analysis. A comparison was also done using another finite element program called VecTor2 which uses a more traditional finite element implementation and has been shown to be accurate [7]. The rectangular elements in VecTor2 are based on linear shape functions and thus requires more than one element across the depth (approximately 10-16 for these beams).

A diagram of the VecTor2 mesh of beam #1358 as well as the results of the mesh sensitivity analysis for beams #131 and #1358 are shown in Figures 5 and 6 respectively. The relationship between time and number of elements was found to be linear. As such, the plots of the Exper./Pred. vs. the elapsed time of the analysis were left out as they are similar to the graphs shown. The results of the mesh sensitivity analysis are summarised in Table 1.

Table 1. Summary of mesh sensitivity analysis beams #131 and #1358.

	# of Elements	*Time* [s]	*Exper./Pred.*
Beam #131			
Augustus-II	88	19	1.14
VecTor2	3340	1665	1.18
Beam #1358			
Augustus-II	49	5	1.57
VecTor2	5032	1074	1.50

As shown by comparing across different platforms, the absolute number of elements does not always imply a superior performance; however, in this case the Augustus-II program runs much faster while also providing similar Exper./Pred. ratios for signicantly fewer elements. In addition, as seen from Figure 6a and Figure 6b, the Augustus-II program converges with very few elements compared to VecTor2. This showcases the robustness of the Augustus-II model with respect to the number of elements compared to the VecTor2 model which typically requires a large number of elements over the depth before converging, and hence leads to a higher computational load. The main advantage of the VecTor2 program is that it provides a detailed analysis of the beam and behaviour at the reinforcement level, which offsets the long computational times of the model, while the Augustus-II program forgoes detailed analysis in favour of a general structural characterization and computational speed.

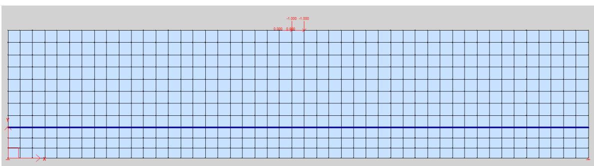
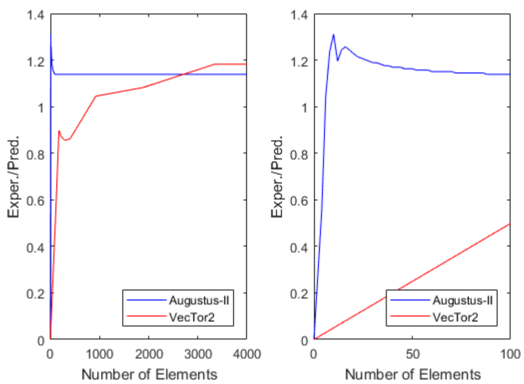
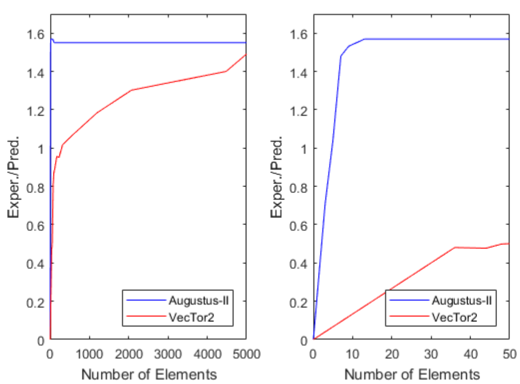


Fig. 5. A mesh of beam #1358 contructed using VecTor2 elements.



(a) Beam #131



(b) Beam #1358

Fig. 6. Graphs of Exper./Pred. vs. number of elements resulting from the mesh sensitivity analyses. Each figure contains two plots with different scales for the number of elements.

Discussion

An average Exper./Pred. ratio of 1.39 shows that the model is consistently under predicting the failure load. This is desirable as an over prediction would be unsafe. A previous study was performed in which various standards for predicting the shear capacity of a beam was compared with the experimental results reported in the two databases [8]. The results are shown in Table 2 alongside the results for the Augustus-II element.

Table 2. Mean Exper./Pred. for both databases calculated using various standards as well as using Augustus-II [8].

	No Stirrups [3]		Stirrups [4]	
	Mean	*CoV*	*Mean*	*CoV*
ACI 318-11	1.42	38.3	1.52	25.8
EC-2	1.10	27.9	1.44	29.6
CSA A23.3-14	1.22	22.3	1.29	1.79
Augustus-II	1.39	21.8	1.39	20.8

While the mean of the Exper./Pred. ratio obtained from Augustus-II is quite large compared to the values calculated using the other standards, particularly EC-2, Augustus-II performed much better when predicting the shear capacity of beams with stirrups. It is also important to note the consistency at which the Augustus-II element is able to predict the failure load of the beams across both databases by showing similar mean Exper./Pred. ratios and coefficient of variation. In contrast, significant differences in the mean Exper./Pred. ratio calculated using the various standards can be seen across the two databases. As standards are used as guidelines for structural design, it is important that they provide a conservative strength estimate so as to prevent catastrophic over-predictions.

The Exper./Pred. ratio was plotted against various parameters in order to investigate their effects on the value. One of the parameter, the a/d ratio, takes the ratio between the shear span (*a*) and the shear depth (*d*) as seen in Figure 3. This parameter can be used as a measure of the slenderness of the beam, with a lower a/d ratio indicating a stockier beam. Below an a/d ratio of approximately 2.5, beam action breaks down and strut-and-tie action becomes dominant. This results in the large spread in the Exper./Pred. values at lower a/d ratios as strut-and-tie action is not captured by the Augutus-II element. A line is drawn across the Exper./Pred. ratio of 1 to indicate how far the predicted value deviates from the reported test value.

From the above plots, certain trends can be seen to result in a more accurate prediction:

- Higher a/d ratio
- Deeper beams

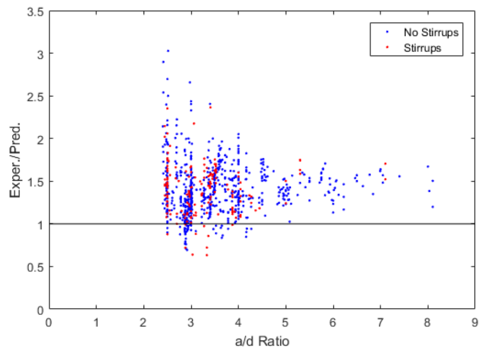


Fig. 7. Graph of the Exper./Pred. vs. a/d ratio for beams with and without shear reinforcement.

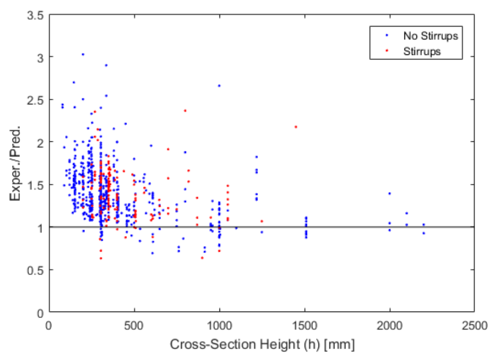


Fig. 8. Graph of the Exper./Pred. vs. cross-section height (h) for beams with and without shear reinforcement.

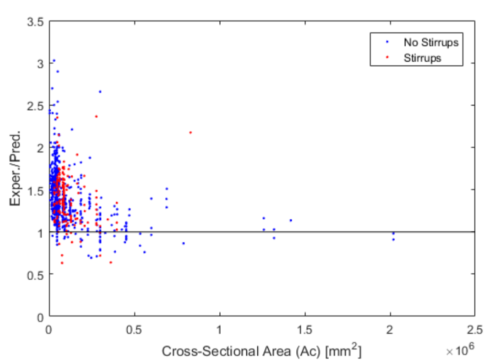


Fig. 9. Graph of the Exper./Pred. vs. cross-sectional area (A_c) for beams with and without shear reinforcement.

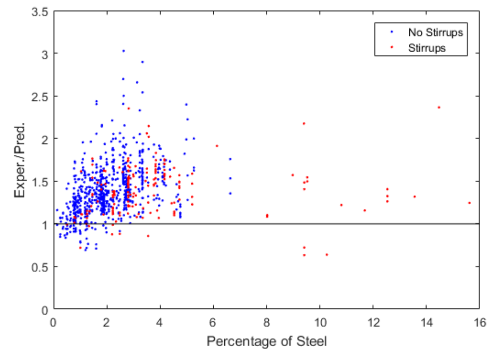


Fig. 10. Graph of the Exper./Pred. vs. percentage of steel for beams with and without shear reinforcement.

- Larger concrete cross-sectional area
- Lower percentage of steel

A contributing factor to the trend between the beam height, cross-sectional area, and the Exper./Pred. ratio is the difficulty in manufacturing smaller beams which constitutes a large subset of the database. As the cross-sectional depth becomes smaller, the reinforcing steel's placement becomes more critical and an offset of a few millimeters can result in substantial changes to the value of the failure load. Additionally, smaller beams amplify the effects of imperfections and material inconsistencies and can result in unrealistic reinforcement which would rarely be found in actual structures. This results in the larger spread found in the smaller beams in Figure 8 and Figure 9. Figure 10 shows a large variance on the Exper./Pred. ratio as the percentage of steel increases. A higher percentage steel runs the risk of over-reinforcing the section, resulting in a failure due to concrete crushing as opposed to the desirable ductile failure. As such, beams found in real-life structures will commonly have a percentage of steel less than 3.5% to avoid over-reinforcing [9]. Therefore, if restricted to beams reinforced with parameters that more closely match those used in practice, the variance of the Exper./Pred. values should decrease. The finite element model handled both beams with and without stirrups with similar performance, as seen in Figure 4.

The following limits were used to evaluate how the results change when restricted to beams which are detailed similar to those found in real-life structures. As these guidelines vary from region to region, the chosen constraints rely mostly on an intuitive subdivision:

- Beam depth: > 1 ft ($\sim$300 mm)
- Cross-Sectional Area: > 1 ft^2 ($\sim$100,000 mm^2)
- Percentage Steel: $< 3.5\%$

Figure 11 shows the distribution of the Exper./Pred. ratio for both the limited group and the excluded group.

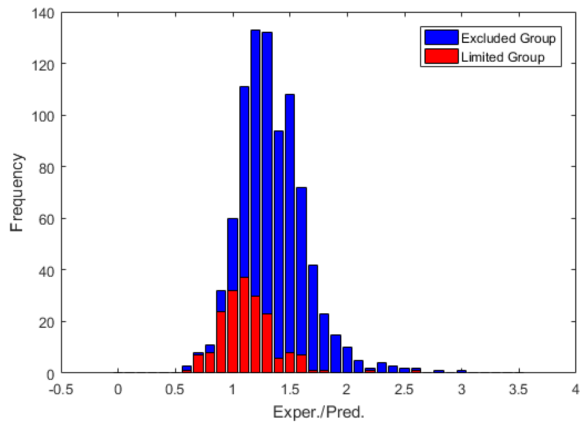


Fig. 11. Exper./Pred. distributions before and after the constraints were applied.

These constraints limit the number of beams to 187 and resulted in an average Exper./Pred. ratio of 1.18 with a coefficient of variation of 21.6%—a more desirable value compared to the average Exper./Pred. ratio of 1.39 for the entire group.

It is difficult to create a model which is both accurate (having a Exper./Pred. ratio close to 1) while also ensuring all analyses are conservative, due to the inherent variability of material strengths and other physical parameters. In addition, the behaviour of concrete is inherently difficult to predict which inevitably leads to a small percentage of beams falling outside of its predicted strength. As such, having a small number of beams with a Exper./Pred. ratio less than 1 is expected. In the case of the two databases used, only 54 beams out of a total of 874 (6%) had unconservative estimates.

Conclusion and Recommendations

The novel 2D reinforced concrete finite element consisting of a shear panel with four surrounding trusses implemented in Augustus-II has been shown to be able to safely predict failure loads of reinforced concrete beams through verification against two databases of point-load beam tests. It produces conservative estimates with an increase in accuracy when limited to beams which are more representative of those found in real-life structures. Additionally, the model was found to converge quickly and was robust with respect to element size—one of the main advantages of the new model. This not only provides an increase in computational speed, but also removes many of the complexities involved with modelling using typical finite elements, such as deciding the number of elements needed along a beam's depth and finding the optimal aspect ratio for the elements.

While the 2D element has now been verified against shear critical beams, more verification is still needed to conclusively prove the accuracy of the model. Currently, work is being done on verifying the model with large-scale structures. Alongside additional testing of the 2D model, steps should be taken to generalize the model into a 3D element, capable of handling both in-plane and out-of-plane loading. This will enable more complex structures, such as nuclear containment silos, to be modeled. As the model is still under development, there is still a need for a better post-processor for the results of the analysis and a more intuitive pre-processor for the creation of the model input files as well. Further study will need to be performed in order to explore whether there is a limit to the beam depth that can be modeled using one element, as well as finding the minimum number of elements required along a beam. The mesh sensitivity analysis performed on the two beams in this paper provides a brief insight into the latter question, but a more rigorous study involving a larger sample size is required to form a conclusive guideline.

Acknowledgement

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